Recent developments in Hodge theory

August 9 – 11, 2023, Sofia, Bulgaria

Aleksandar Petkov, Sofia University and Institute of Mathematics and Informatics, Sofia

Title: ***The Yamabe problem in quaternionic contact geometry***

Abstract: We are going to consider in this talk the quaternionic contact (qc) version of the well known Yamabe problem in Riemannian geometry. It is shown that it has a solution on any compact qc manifold which is non-locally qc equivalent to the standard 3-Sasakian sphere. Namely, it is proved that on a compact non-locally spherical qc manifold there exists a qc conformal qc structure with constant qc scalar curvature.

The talk is based on a joint work with Stefan Ivanov.

Christopher Brav, HSE University, Moscow, Russia

Title: ***An introduction to solid algebraic geometry***

Abstract: Solid algebraic geometry in the sense of Clausen-Scholze is a generalization of algebraic geometry in which the basic test spaces are spectra of solid analytic rings (a certain generalization of Huber pairs from odic geometry). After giving some basic definitions and emphasizing new phenomena, we discuss a selection of applications, from Beilinson-Parshin adelic resolutions of quasi-coherent sheaves and to the study of infinite dimensional algebraic geometry.

This is joint work with Artem Prikhodko and Grigory Konovalov.

Gueo Grantcharov, Florida International University, USA

Title: ***Borel-Weil Theorem and Laplace eigenfunctions on Riemannian symmetric spaces***

Abstract: In this talk I'll present a geometric relation between the Laplace-Beltrami spectra and eigenfunctions on compact Riemannian symmetric spaces and the Borel-Weil theory using ideas from symplectic geometry and geometric quantization. The examples of the classical simply-connected spaces of rank one and the space $SU(3)/SO(3)$ will be considered in which one can describe all eigenfunctions in terms of harmonic polynomials.

This is joint work with D. Grantcharov and C. Montoya.

Eder Correa, University of Campinas - UNICAMP

Title: DHYM connections on higher rank homogeneous vector bundles

Abstract: The deformed Hermitian Yang-Mills (dHYM) equation is a special Lagrangian type condition in complex geometry. This equation was independently discovered around 2000 by Mariño-Minasian-Moore-Strominger and Leung-Yau-Zaslow. Since then, it has been extensively studied by both physicists and mathematicians because of its relevance to gauge theory, quantum field theory, and algebraic geometry. In this talk, some results related to the construction of dHYM connections on higher rank homogeneous vector bundles over rational homogeneous varieties will be discussed. The main goal is to present examples of dHYM connections on slope unstable holomorphic vector bundles.

Ernesto Lupercio, Cinvestav-IPN

Title: TBA

Abstract:

Josef Svoboda, University of Miami, USA

Title: TBA

Abstract:

Lino Grama, University of Campinas – UNICAMP

Title: Twisted Kähler-Einstein metrics on flag varieties

Abstract: In this talk, we describe invariant twisted Kähler-Einstein (tKE) metrics on flag varieties. We also explore some applications of the ideas involved to provide a precise description for the greatest Ricci lower bound of an arbitrary Kähler class on a flag variety. By means of this description, we establish some inequalities related to optimal volume upper bounds for Kähler metrics just using tools from Lie theory. Further, we describe the set of tKE metrics for several examples, the projectivization of the tangent bundle of $\mathbb{P}^{n+1}$, and families of flag varieties with Picard number 2.

This is a joint work with Eder Correa.

Ludmil Katzarkov, University of Miami and Institute of Mathematics and Informatics, Sofia

Title: ***A, B structures and applications***

Abstract: In this talk we introduce new Hodge type birational invariants. Examples will be considered.

Misha Shkolnikov, Institute of Mathematics and Informatics, Sofia

Title: ***Tropical Caustics***

Abstract: I will tell about our attempts to design tropical versions of wave fronts and their caustics, originally studied in optics, focusing on the case of an internal propagation from a boundary of a convex domain on the plane with an integral affine structure for which a tropical caustic curve is a piecewise linear graph inside the domain. Since our approach is naturally invariant under automorphisms of the cotangent square lattice, this setup is intimately related to the intrinsic geometry of symplectic toric surfaces and gives an intriguing perspective on their moduli space. For example, singularities of tropical caustic curves, that we classify completely, encode explicit polyhedral strata in this space, and the lengths of edges of caustics with a given singularity type give exact coordinates on the corresponding stratum.

The talk is based on joint projects with Grigory Mikhalkin, Nikita Kalinin, Ernesto Lupercio and Daniel Tabares.

Morgan Brown, University of Miami, USA

Title: ***The Dual Complex of a semi-log canonical Surface***

Abstract: Semi-log canonical varieties are a higher-dimensional analogue of stable curves. A stable curve has associated with it a dual graph, with vertices corresponding to the irreducible components and edges to the nodes of the curve. Moreover, if the curve is a divisor in a surface, the PL type of this graph is unchanged if we replace the pair by its dlt minimal model. I will present a construction to produce a two-dimensional complex from a semi-log canonical surface Δ that recovers the PL type of a dlt minimal model when Δ is the boundary of a threefold pair (X, Δ).

Phillip Griifiths, Institute for Advanced Study, USA

Title: ***Period mappings for anti-canonical pairs****

Abstract: Anti-canonical pairs (Y, D) are logarithmic *K3* surfaces. It is well known that they have a rich geometry. A recent result, whose proof was motivated by mirror-symmetry, establishes a conjecture by Looijenga giving conditions for smoothability of the cusp obtained by contracting D. A central ingredient in the proof is a global Torelli theorem using the mixed Hodge structure on $H^2(Y - D)$. In this talk we will formulate and sketch the proof of this result.

*Based on the works of Looijenga [L81], Friedman [F16], Engel-Friedman [EF21], and Gross-Hacking-Keel [GHK15]. Presentation including notations, largely follows [F16].

Richard Paul Horja, University of Miami, USA

Title: ***Secondary spherical functors***

Abstract: We will present a conjectural construction of the web of spherical functors associated to the secondary polytope defined by Gelfand, Kapranov and Zelevinsky in toric geometry. Some K-theoretic supporting evidence will be discussed.

This is joint work with Ludmil Katzarkov.

Robert Stephen Cantrell, University of Miami, USA

Title: ***Resource Matching in Spatial Ecology and Evolutionary Advantage***

Abstract: A convergence of concepts from game theory, ecological theory and mathematics combine to explain how resource matching can convey evolutionary advantage. In this talk we discuss the historical emergence of the this convergence of concepts and how it consequently leads to predictions of evolutionary advantage in spatially heterogeneous and either temporally constant or seasonal habitats. In the case of seasonal habitats we touch upon some current work on the impact of memory and perception in migration.

Rodolfo Aguilar, University of Miami, USA

Title: ***On nilpotent quotients of normal quasi-projective groups***

Abstract: We show that if X is a smooth complex quasi-projective manifold the quasi-Albanese map of which is proper, then the torsionfree nilpotent quotients of $\pi_1(X)$ are the same ones as those of the Stein factorisation of its quasi-Albanese image. This implies that the étale Galois cover of X associated to the nilpotent completion of $\pi_1(X)$ is holomorphically convex. This last result is proved in the quasi-projective case by 3 other methods by Green-Griffiths-Katzarkov, which motivated the present work.

Joint work with F. Campana.

Velichka Milousheva, Institute of Mathematics and Informatics, Sofia

Title: ***Canonical Parameters on Timelike Surfaces with Parallel Normalized Mean Curvature Vector Field in the Minkowski 4-space***

Abstract: We introduce special isotropic parameters (which we call canonical parameters) for the class of timelike surfaces with parallel normalized mean curvature vector field in the Minkowski 4-space. These parameters allow us to describe this class of surfaces in terms of three geometrically determined functions. We prove Fundamental theorem stating that any timelike surface with parallel normalized mean curvature vector field is determined uniquely up to a motion by three functions satisfying a system of three partial differential equations.

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Vestislav Apostolov, Université du Québec à Montréal, Canada and Institute of Mathematics and Informatics, Sofia

Title: ***The generazed Calabi-Yau problem***

Abstract: I will describe an extension, proposed by Hitchin and Gualtieri, of the notion of a Calabi-Yau structure to generalized Kähler geometry. I will then discuss a conjectural classification of the generalized Kähler Calabi-Yau geometries, expressed in terms of Bogomolov-Beauville decomposition, and present a partial resolution.

This talk is based on a joint work with X. Fu, J. Streets and Y. Ustinovskiy.

Vladimir Mitankin,

Title: ***Brauer-Manin obstruction for Campana orbifolds***

Abstract: In this talk I shall explain the notion of Campana points on orbifold pairs and how they generalise both the theories of integral and rational points on algebraic varieties. I will construct an adelic space for Campana points which will allow me to define local-global principles for them. I will then explain a Campana version of the Brauer-Manin obstruction interpolating between Manin's classical version for rational points and the integral version developed by Colliot-Thélène and Xu. Finally, I will apply this obstruction theory to study the status of Campana local-global principles for quadric orbifold pairs with linear divisor. This talk is based on a joint work with Masahiro Nakahara and Sam Streeter.